

Hong Kong Mathematics Olympiad (2017/18)  
Final Event 1 (Individual)

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>	Team No.	<input type="text"/>
			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
						<input type="text"/>		

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.  
除非特別聲明，答案須用數字表達，並化至最簡。

1. Given that  $x^2 = y^2 - 4y$ , where  $x$  and  $y$  are integers. Determine the largest value of  $A = x + y$ .

已知  $x^2 = y^2 - 4y$ ，其中  $x$  及  $y$  為整數。求  $A = x + y$  的最大值。

$A =$

2. Given that  $y = \sqrt{9A^2 - 12A + 4} \pm \sqrt{A^2 - 4A + 4} \pm \sqrt{A^2 + 6A + 9}$  and  $B$  is the least positive value of  $y$ , determine the value of  $B$ .

已知  $y = \sqrt{9A^2 - 12A + 4} \pm \sqrt{A^2 - 4A + 4} \pm \sqrt{A^2 + 6A + 9}$ ，且  $B$  是  $y$  的最小正數值，求  $B$  的值。

$B =$

3. Let  $C$  be a positive integer. Given that  $144 + (B+1)^C$  is a perfect square, determine the value of  $C$ .

設  $C$  為正整數。已知  $144 + (B+1)^C$  為平方數，求  $C$  的值。

$C =$

4. Given that  $x + \frac{1}{x} = C$ , determine the value of  $D = x^3 + \frac{1}{x^3}$ .

已知  $x + \frac{1}{x} = C$ ，求  $D = x^3 + \frac{1}{x^3}$  的值。

$D =$

Hong Kong Mathematics Olympiad (2017/18)  
Final Event 2 (Individual)

FOR OFFICIAL USE

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			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
						<input type="text"/>		

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除非特別聲明，答案須用數字表達，並化至最簡。

1. Determine the value of  $a$ , where  $a$  is the sum of all the digits of  $7778^2 - 2223^2$ .  
 $7778^2 - 2223^2$  之值的所有數字之和是  $a$ ，求  $a$  的值。

$a =$

2. If the number of trailing zeros of the product  $a \times (a-1) \times (a-2) \times \cdots \times 2 \times 1$  is  $b$ , determine the value of  $b$ .

$$a \times (a-1) \times (a-2) \times \cdots \times 2 \times 1 = \cdots * \underbrace{000 \dots 0}_{\text{the number of "0"s is } b}$$

\* represents non-zero digit

若  $b$  是乘積  $a \times (a-1) \times (a-2) \times \cdots \times 2 \times 1$  的尾隨零的數量，求  $b$  的值。

$$a \times (a-1) \times (a-2) \times \cdots \times 2 \times 1 = \cdots * \underbrace{000 \dots 0}_{\text{"0"的數量是 } b}$$

\* 代表非零個位數

$b =$

3. If  $c$  is the remainder of  $2^{10} - 2^8 + 2^6 - 2^4 + 2^2$  divided by  $b$ , determine the value of  $c$ .

若  $c$  是  $2^{10} - 2^8 + 2^6 - 2^4 + 2^2$  除以  $b$  的餘數，求  $c$  的值。

$c =$

4. Determine the value of integer  $d$ , so that  $x^{13} + cx + 90$  is divisible by  $x^2 - x + d$  for any real number  $x$ .

求整數  $d$ ，使得對於任何實數  $x$ ， $x^{13} + cx + 90$  可被  $x^2 - x + d$  整除。

$d =$

Hong Kong Mathematics Olympiad (2017/18)  
Final Event 3 (Individual)

FOR OFFICIAL USE

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							Min.	Sec.
						<input type="text"/>		

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除非特別聲明，答案須用數字表達，並化至最簡。

1. Given that  $A = (3a - X)^2 + (3b - X)^2 + (3c - X)^2 + 6$ . If  $a$ ,  $b$  and  $c$  are real numbers,  $X = a + b + c$  and  $X^2 = a^2 + b^2 + c^2$ , determine the least value of  $A$ .  
已知  $A = (3a - X)^2 + (3b - X)^2 + (3c - X)^2 + 6$ 。若  $a$ 、 $b$  及  $c$  為實數， $X = a + b + c$  及  $X^2 = a^2 + b^2 + c^2$ ，求  $A$  的最小值。

$A =$

2. Suppose that there are  $A$  boys and  $30 - A$  girls in a class. If the average weight of the boys is 60kg, the average weight of the girls is 45kg, and the average weight of the students in the class is  $B$  kg, determine the value of  $B$ .  
假設班中有  $A$  名男同學及  $30 - A$  名女同學。若男同學的平均體重為 60kg，女同學的平均體重為 45kg 及全班同學的平均體重為  $B$  kg，求  $B$  的值。

$B =$

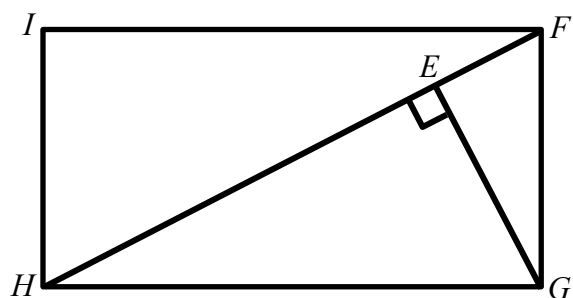
3. If  $n$  is a positive integer,  $a_1 = B$ , and  $a_{n+1} = \begin{cases} \frac{a_n}{2} & \text{if } a \text{ is even} \\ 3a_n + 1 & \text{if } a \text{ is odd} \end{cases}$ , determine the value of  $C = a_{2018}$ .

若  $n$  是正整數、 $a_1 = B$  及  $a_{n+1} = \begin{cases} \frac{a_n}{2} & \text{若 } a \text{ 是偶數} \\ 3a_n + 1 & \text{若 } a \text{ 是奇數} \end{cases}$ ，求  $C = a_{2018}$  的值。

$C =$

4. Suppose that a rectangle  $FGHI$  is divided into two right-angled triangles by line  $FH$ . The triangle  $\triangle FGH$  is then divided into two right angled-triangles by line  $EG$ . If the ratio of the lengths  $FH : FG$  is  $C : 1$  and the ratio of the areas of  $\triangle EGH$  to  $\triangle FEG$  is  $D : 1$ , determine the value of  $D$ .

長方形  $FGHI$  被直線  $FH$  分為兩個直角三角形。三角形  $\triangle FGH$  則被直線  $EG$  分為另外兩個直角三角形。若  $FH : FG = C : 1$  及三角形  $\triangle EGH$  與三角形  $\triangle FEG$  的面積比為  $D : 1$ ，求  $D$  的值。



$D =$

Hong Kong Mathematics Olympiad (2017/18)  
Final Event 4 (Individual)

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除非特別聲明，答案須用數字表達，並化至最簡。

1. If  $a$  is the remainder of  $(1^{2018} + 2^{2018} + 3^{2018} + 4^{2018}) \div 5$ , determine the value of  $a$ .  
若  $a$  為  $(1^{2018} + 2^{2018} + 3^{2018} + 4^{2018}) \div 5$  的餘數，求  $a$  的值。

$a =$

2. If  $x, y$  are positive integers and  $b$  is the number of ways of  $x, y$  such that the product  $x \times y = \overline{1aa}$ , determine the value of  $b$ .  
若  $x, y$  為正整數及  $b$  為  $x, y$  組合的數量使得它們的乘積  $x \times y = \overline{1aa}$ ，求  $b$  的值。

$b =$

3. If  $xyz + xy + xz + yz + x + y + z + 1 = 30b + 87$  for positive integers  $x > y > z$ , determine the value of  $c = x + y + z$ .  
若對於正整數  $x > y > z$ ， $xyz + xy + xz + yz + x + y + z + 1 = 30b + 87$ ，求  $c = x + y + z$  的值。

$c =$

4. Let  $d$  be the area (in  $\text{cm}^2$ ) of a rectangle that can be tessellated by square tiles with side length of  $\frac{c}{3}$  cm. If the rectangle can also be tessellated by rectangular tiles with width of  $\frac{c}{2}$  cm and length of 7 cm, determine the least positive value of  $d$ .

設  $d$  為某長方形的面積(平方厘米)，它能被一種為邊長  $\frac{c}{3}$  厘米的正方形階磚密鋪。若該長方形亦能被一種為闊  $\frac{c}{2}$  厘米、長 7 厘米的長方形階磚密鋪，求  $d$  的最小正數值。

$d =$